## 10.7) Power Series:

- After studying the convergence and divergence of series, we will now study the sum of infinite polynomials. we call these sum as "power series".

### * Definition:

$$\left\{ \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2, \ldots \right\}$$

where the center $c$ and the coeffeciants $a_0, a_1, a_2, \ldots$ are constants

### * Geometric Power Series:

- The geometric power series is a special type of power series where $x$ is centred at $0$ $(c=0)$ and coeffeciants are all $1$:

$$\left\{ \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots x^n \right\}$$

- if $|x| < 1 \longrightarrow \left\{ S = \dfrac{1}{1-x} \right\}$

## * Radius of Convergence:

- in other words, the radius of convergence is like asking for what values for $x$ will the series converge?:

- For a geometric series, we stated that

$$\sum_{n=0}^{\infty} x^n \text{ converges}$$

iff $|x| < 1$:

- so radius of convergence is:

$$\{-1 < x < 1\}$$

## * Determining radius of convergence:

- To determine the radius of convergence either apply "Ratio Test" or "Root Test"

- after finding the interval, we check if the boundary is included by subbing boundary points on the series, and determining if it converges on boundary aswell

- ex, : $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

- Determine radius of convergence and interval of convergence:

- apply root test:

$$\left| \frac{x^n}{n} \right|^{\frac{1}{n}} = |x|$$

- $|x| < 1$ so Radius $= 1$

- Interval

$$\{-1 < x < 1\}$$

- Now Lets test boundaries :

- sub $x = -1$ in the series :

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(-1)^n}{n} = \sum \frac{(-1)^{2n-1}}{n}$$

- $(-1)^{odd} = -1$ and since $2n-1$ is odd $\rightarrow$ $\sum_{n=1}^{\infty} -\frac{1}{n}$ diverges by $\int$ test

- sub $x = 1$:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1)^{n}}{n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$ Converges by A.S.T.

$\therefore$ Interval of convergence:

$$\boxed{-1 < x \leq 1}$$

- ex$_2$: $\sum_{n=0}^{\infty} n!\, x^n$

- apply root test:

$$|n!\, x^n|^{\frac{1}{n}} = \infty \cdot |x| > 1 \quad \forall x \in \mathbb{R} - \{0\}$$

$\therefore$ series diverges

* Series multiplication Theorem:

$$\left(\sum_{n=0}^{\infty} a_n x^n\right) \cdot \left(\sum_{n=0}^{\infty} b_n x^n\right) = \sum_{n=0}^{\infty} C_n x^n$$

- ex: $\left(\sum x^n\right) \cdot \left(\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}\right)$

$$= \left(1 + x + x^2 + \cdots\right)\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right)$$

$$= 1\left(x - \frac{x^2}{2} + \frac{x^3}{3},,,,\right) + x\left(x - \frac{x^2}{2} + \frac{x^3}{3},,,\right) + x^2\left(x - \frac{x^2}{2} + \frac{x^3}{3},,\right),,,,$$

$$\left(x - \frac{x^2}{2} + \frac{x^3}{3},,,,\right) + \left(x^2 - \frac{x^3}{2} + \frac{x^4}{3},,,\right) + \left(x^3 - \frac{x^4}{2} + \frac{x^5}{3},,,\right),,,.$$

. sort them in order:

$$x + \frac{x^2}{2} + \frac{5x^3}{6} - \frac{x^4}{6},,,,$$